

Quantitative Methods in Economics Self-Administered Proficiency Examination

Harvard Summer School

The Harvard Summer School Economics Program offers a variety of courses, ranging from introductory economics to advanced classes in micro- and macroeconomic theory, econometrics, international economics, and finance. Some of these courses require students to have analytical and technical facility before registering. This is indicated in the course Prerequisites (“must pass proficiency examination”) in the [Summer School Course Catalog](#).

All students registering for economics courses that require the proficiency exam, as indicated in the Prerequisites, must self-administer and pass the exam. (Harvard College students should see the relevant section in the Summer School Catalog for term-time courses that exempt them from the proficiency examination.)

Suggested texts providing useful review of this material are *Fundamental Methods of Mathematical Economics*, 3rd ed., by Alpha Chiang (New York: McGraw-Hill, 1984) and *An Introduction to Mathematical Statistics and Its Applications*, 2nd ed., by Richard J. Larsen and Morris L. Marx (Englewood Cliffs, NJ: Prentice Hall, 1986). For a review of the fundamental tools of economic analysis, refer to the relevant chapters in such texts as *Economics*, 9th ed., by Baumol and Blinder (New York: Harcourt Brace Jovanovich, 2003), *Economics*, 13th ed., by Lipsey, Steiner and Purvis (New York: Harper & Row, 2007), or *Principles of Economics*, 5th ed., by N. Gregory Mankiw (Fort Worth, TX: Harcourt College Publishers, 2008).

**Economics Proficiency Exam
Harvard Summer School**

Instructions

1. There are ten questions on the exam. Some of these questions are broken into separate parts, so there are 29 answers you will be asked to supply in all.
2. You may work for up to 60 minutes on the exam. Please time yourself. Write all answers on a separate sheet of paper.
3. Calculators are prohibited. Give your answer in the neatest, simplest form possible. Please reduce fractions if possible. Carefully label all graphs.
4. Please use the answer key provided at the end to grade your exam. A passing grade is a minimum of 12 correct answers out of a possible 29.

Exam Questions

1. Mr. B has \$20.00 to spend on fruit. Apples cost \$0.25 apiece. Oranges cost \$0.50 apiece.
 - a. Write down a formula for the maximum number of apples Mr. B can buy as a function of the number of oranges he purchases.
 - b. In a graph with the quantity of oranges measured on the horizontal axis and the quantity of apples measured on the vertical axis, indicate the area that represents all the combinations of apples and oranges that Mr. B can afford to purchase.
2. Graph the equation $y = 30 - x^2 + 2x$.
3. Ms. Marksman is standing in the x-y plane at the coordinates (50,70). She shoots a bullet through a paper target located at (200,10). The bullet travels in a straight line, hits the target, and continues traveling at a constant speed.
 - a. Write down the formula for the straight line within which the bullet travels.
 - b. If the bullet hits the target six seconds after Ms. Marksman fires her gun, what will be the bullet's coordinates one second after passing through the target?
4. Mr. Farmer has a triangular garden with corners at (0,0), (50,20) and indicates a distance in meters from the (200,0). Each coordinate indicates a distance in meters from the origin. If it costs \$2 to irrigate one square meter of a garden for one week, what will be Mr. Farmer's total irrigation bill for the ten-week dry season?
5. Suppose that demand in the market for shirts is given by the formula $p = 100 - 2Q_a$ where p is the price per shirt, Y is aggregate weekly consumer income, and Q_a is the quantity of shirts demanded per week. (Continued next page.)

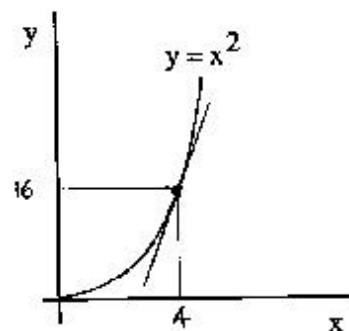
Suppose that the supply for shirts is given by

$$p = 20 + 6 \cdot Q_s$$

where Q_s is the quantity of shirts supplied per week.

- At what price is the quantity supplied equal to the quantity demanded?
- When quantity supplied equals quantity demanded, what is the total weekly spending on shirts in this market?
- Graph the supply curve and the demand curve together in a single diagram with price on the vertical axis and quantity on the horizontal axis. Indicate the price at which $Q_s = Q_d$. Indicate the total spending on shirts.
- Suppose the price of shirts is set by law at $p = 50$. Calculate the difference between the quantity demand and quantity supplied at that price.

6. What is the slope of the line tangent to the curve $y = x^2$ at the point $(4, 16)$?



7. Calculate the slope of the line tangent to the curve $y = F(x)$ when

- | | | |
|----|---------------------------------------|-----------|
| a. | $F(x) = \ln x,$ | $x = 1/8$ |
| b. | $F(x) = 4x + 25$ | $x = 9$ |
| c. | $F(x) = \frac{\sqrt{x}}{9}$ | $x = 1/9$ |
| d. | $F(x) = 5e^{2x}$ | $x = 0$ |
| e. | $F(x) = \frac{2}{x}$ | $x = -4$ |
| f. | $F(x) = \frac{1}{6}x^3 + \sqrt[3]{x}$ | $x = 8$ |

8. Calculate $\frac{d^2y}{dx^2}$ when

- a. $y = 4x^4 + 3x^2 + 8$
- b. $y = 8 \ln x$
- c. $y = x^{0.25}$

9. Let $y = \sqrt{x_1 \cdot \ln x_2}$. Assume x_1 and x_2 take only positive values. Calculate each of the following partial derivatives:

- a. $\frac{\partial y}{\partial x_1}$
- b. $\frac{\partial^2 y}{\partial x_1^2}$
- c. $\frac{\partial y}{\partial x_2}$
- d. $\frac{\partial^2 y}{\partial x_2^2}$
- e. $\frac{\partial^2 y}{\partial x_2 \partial x_1}$
- f. $\frac{\partial^2 y}{\partial x_1 \partial x_2}$

10. Suppose that consumer spending plans are described by the function

$$C = c_0 + c_1 \cdot Y$$

where C is total consumer spending, Y is income and c_0 and c_1 are positive constants, and $c_1 < 1$. Suppose also that business investment spending plans are given by the function

$$I = i_0 - i_1 \cdot r$$

where I is total business investment spending, r is the interest rate and i_0 and i_1 are positive constants. Finally, suppose that the economy is in equilibrium when total planned spending equals total income, that is when

$$Y = C + I$$

- a. Find an expression for the economy's equilibrium income level.
- b. Find an expression that describes how equilibrium income changes as i_0 varies.
- c. Graph equilibrium income as a function of the interest rate in a graph with r on the vertical axis and Y on the horizontal. What is the slope of this function?

**Short Answers to Sample Questions
for the Proficiency Exam**

Harvard Summer School Economics Program

Note: References in parentheses are to relevant pages in two standard texts:

Chiang, Alpha, Fundamental Methods of Mathematical Economics (3rd ed.)

Larsen, Richard and Morris Marx, An Introduction to Mathematical Statistics and Its Applications
(2nd ed.) [Designated as L&M]

1. a. $A = 80 - 2O$ where A = apples, O = oranges
b. Graph is a straight line with vertical intercept 80, horizontal intercept 40; area of all affordable bundles is entire area between this line and the two axes.
(Chiang, p. 24)
2. Graph is a concave parabola with maximum at $x = 1, y = 31$ (Chiang, p. 41)
3. a. $y = 90 - 2/5 x$ 4. \$40,000 5. a. $P^* = 80$
b. (225, 0) b. Since $Q^* = 10, P^*Q^* = 800$
(Chiang, p. 74) d. $Q_2 - Q_3 = 20$
(Chiang, pp. 46-51)
6. $dy/dx = 8$ at (4, 16) (Chiang, p. 131)
7. a. $dF/dx = 1/x$; at $x = 1/8, dF/dx = 8$ (Chiang, p. 292)
b. $dF/dx = 4$ at all x (Chiang, p. 155)
c. $dF/dx = 1/18 x^{1/2}$; at $x = 1/9, dF/dx = 1/6$ (Chiang, p. 156)
d. $dF/dx = 10e^{2x}$; at $x = 0, dF/dx = 10$ (Chiang, p. 293)
e. $dF/dx = -2x^{-3}$; at $x = -4, dF/dx = -1/8$ (Chiang, p. 156)
f. $dF/dx = 1/2 x^2 + 1/3 x^{-23}$; at $x = 8, dF/dx = 32 1/12$ (Chiang, p. 156)
8. a. $d^2y/dx^2 = 48x^2 + 6$
b. $d^2y/dx^2 = -8x^2$
c. $d^2y/dx^2 = .1875x^{1.75}$
(Chiang, p. 239)

$$9. \quad a. \quad \frac{\partial y}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} (\ln x_2)^{\frac{1}{2}} \qquad d. \quad \frac{\partial^2 y}{\partial x_2^2} = x_1^{\frac{1}{2}} \left[-\frac{1}{4} (\ln x_2)^{-\frac{3}{2}} \frac{1}{x_2} + \frac{1}{2} (\ln x_2)^{-\frac{1}{2}} \left(-\frac{1}{x_2} \right) \right]$$

$$b. \quad \frac{\partial^2 y}{\partial x_1^2} = -\frac{1}{4} x_1^{-\frac{3}{2}} (\ln x_2)^{\frac{1}{2}} \qquad e. \quad \frac{\partial^2 y}{\partial x_2 \partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} \left[\frac{1}{2} (\ln x_2)^{-\frac{1}{2}} \frac{1}{x_2} \right]$$

$$c. \quad \frac{\partial y}{\partial x_2} = x_1^{\frac{1}{2}} \left[\frac{1}{2} (\ln x_2)^{-\frac{1}{2}} \frac{1}{x_2} \right] \qquad f. \quad \frac{\partial^2 y}{\partial x_1 \partial x_2} = \frac{1}{2} x_1^{-\frac{1}{2}} \left[\frac{1}{2} (\ln x_2)^{-\frac{1}{2}} \frac{1}{x_2} \right]$$

(Chiang, p. 174)

10. a. $Y^* = \frac{1}{1-c_1}(c_0+i_0-i_1r)$, (Chiang, p. 52)

b. $\frac{\partial Y^*}{\partial i_0} = \frac{1}{1-c_1}$, the multiplier (Chiang, p. 52)

c. Graph is straight line with vertical intercept $r = \frac{c_0+i_0}{i_1}$ and horizontal intercept $Y^* = \frac{c_0+i_0}{1-c_1}$.

The slope is given by $\frac{dr}{dY^*} = \frac{c_1-1}{i_1}$.